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Stochastic theory of log-periodic patterns

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Abstract. We introduce an analytical model based on birth–death clustering processes to help in understanding the empirical log-periodic corrections to power law scaling and the finite-time singularity as reported in several domains including rupture, earthquakes, world population and financial systems. In our stochastic theory log-periodicities are a consequence of transient clusters induced by an entropy-like term that may reflect the amount of co-operative information carried by the state of a large system of different species. The clustering completion rates for the system are assumed to be given by a simple linear death process. The singularity at t_0 is derived in terms of birth–death clustering coefficients.

1. Overview

Increasing evidence of accelerated patterns having an overall power law behaviour with superimposed log-periodic oscillations has been found in a variety of applied domains. These observations have been reported in a series of experiments on rupture in heterogeneous media [1, 2] and from the historical data analysis of earthquakes [3–6], world population [7] and financial stock markets [8–13] (see the examples in figure 1). It has been also argued that log-periodic corrections to scaling should be present in a wider class of out-of-equilibrium dynamical systems (see, for example, [14, 15]). The logarithmic modulations are periodic in $t - t_0$ and not on t and are precursors to a spontaneous finite-time singularity t_0 at which they accumulate.

The interest in log-periodic corrections to power law scaling is twofold. On the one hand they enhance the fit quality to observed data with better precision than simple power laws by adjusting the (frequency, local minima and maxima of) oscillations. On the other hand, their real-time monitoring could, in principle, allow for an enhancement of predictions in different contexts [15, 16].

At the theoretical level, log-periodic oscillatory structures have been associated to the existence of complex fractal dimensions [9] and critical exponents [15–18]. However, as pointed out in [19], predictions of stock market crashes using complex critical exponents should be taken with some care, not only because of the many fitting parameters required but also because the time period used to perform the fit is rather long [20]. This does not mean that the apparent acceleration and the log-periodic modulation do not actually exist—the whole subject deserves further investigation.

Most recently, log-periodic patterns associated with financial crashes have also been shown to stem from models for stock markets inspired by percolation phenomena [21–23]. Furthermore, logarithmic oscillations have been found in an off-lattice bead–spring model of

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a polymer chain in a quenched porous medium under the influence of an external field [24] and in a uniform spin model on a fractal [25]. However, apart from these simulation studies, there is no a convincing microscopic theoretical model which substantiates the idea that the singularity at finite t_0 (for example, a financial crash) is a critical point. Also, there is as yet no a fundamental theory that substantiates the claim for the precursory, universal log-periodic oscillations on large time scales.

In this paper we attempt to provide a new scenario within which to elaborate an analytical microscopic theory and contribute towards an understanding of the underlying physics of log-periodic patterns. We introduce a stochastic model based on birth–death clustering processes to sustain the claim of log-periodic corrections to scaling and of a finite-time singularity. In our theory, transient clusters are formed following an entropy-like formula that may reflect the amount of co-operative information (or disorder) carried by the state of a large system of different species. The clustering completion rates for the system are assumed to be exponentially distributed according to a simple linear death process. The singularity at t_0 is derived in terms of birth–death clustering coefficients.

2. Stochastic theory

We write the governing equation for power law behaviour decorated by large-scale log-periodic oscillations as a superposition of two terms

$$G(t) = G_0(t) + G_\infty(t) \tag{1}$$

where G_0 takes the form of a pure power law and G_∞ represents the (universal periodic) corrections.

Similarly to [1, 3], the first term is taken to be

$$\frac{\mathrm{d}G_0(t)}{\mathrm{d}t} = \kappa(t_0 - t)^{\alpha-1} \tag{2}$$

with t_0 a finite time at which a singularity appears and the exponent α satisfies $\alpha \neq 1$. Integration of this equation yields

$$G_0(t) = G_0(t_0) - \frac{\kappa}{\alpha}(t_0 - t)^{\alpha}. \tag{3}$$

In the following we seek for an approximate form to the correction term G_∞.

2.1. Galerkin finite-element method for $G_N(s, t)$

The starting point of our theory is to assume that the two-dimensional (for example, energy- and time-dependent) G_N function of a discretized system of N nodes is the solution of some nonlinear differential equation (for example, a diffusion equation with particular boundary conditions) which we do not know but which we shall derive an answer for.

Using the standard Galerkin finite-element method described in the appendix (see also, for example, [26]), a general trial solution to this unknown differential equation can be approximated as

$$G_N(s, t) \equiv \sum_{j=0}^{N} g_j(s)\tilde{P}_j(t) \tag{4}$$

where g_j are basic (interpolation) functions, and $\tilde{P}_j$ are the so-called test functions. The terms $g_j(s)$ are often referred to as trial functions and equation (4) as the trial solution at nodal points.

Without loss of generality, for all t and different j-states we can rewrite $\tilde{P}$ in a more suitable form as the sum of even and odd parts:

$$\tilde{P}_j(t) = p_{2j}(t) + p_{2j+1}(t) \qquad (j = 0, 1, 2, \ldots, N). \tag{5}$$

This means that $\tilde{P}_0 = p_0 + p_1$, $\tilde{P}_1 = p_2 + p_3$, $\tilde{P}_2 = p_4 + p_5, \ldots$.

Hence, in the limit $N \to \infty$, we then get

$$\sum_{j=0}^{\infty} p_j(t) = \sum_{j=0}^{\infty} \tilde{P}_j(t) \equiv 1. \tag{6}$$

This result will prove useful later when we associate the test functions $\tilde{P}_j$ with the equilibrium state probabilities to be characterized by birth–death processes.

2.1.1. Birth–death model for $p_j(t)$—effect of disorder. The test functions $\tilde{P}_j(t)$ are usually determined by solving a system of differential equations (in time) generated by some governing equation, and if N is made arbitrarily large the error introduced becomes small (see the appendix). In order to gain insight into the dynamics leading to log-periodic structures, we next take a different approach and relate the test functions to a large number of processes forming clusters or aggregates that change as a function of time (for example, cell populations, customers queueing, interactive multi-agent ensembles, investors groups) acting collectively to pass on information or to introduce system disorder.

We assume that stochastic 'birth' and 'death' clustering processes occur according to a simple one-dimensional *birth-and-death* model (see, for example, [27]). The state probabilities $p_j(t)$ in this case are obtained recursively from

$$\hat{\lambda}_j(t) p_j(t) = \hat{\mu}_{j+1}(t) p_{j+1}(t) \qquad (j = 0, 1, 2, \ldots). \tag{7}$$

By a choice of the birth coefficients $\hat{\lambda}_j > 0$ and of the death coefficients $\hat{\mu}_j > 0$, various stochastic models can be constructed (for example, queueing models in which costumers correspond to the 'population', arrivals are 'births' and departures are 'deaths'). In other words, the quantity $\hat{\lambda}$ is interpreted as the birth rate and $\hat{\mu}$ the death rate when the population is at the state j.

Equation (7) together with the normalization condition of equation (6) can easily be solved to yield the following statistical-equilibrium state distribution (as seen from an arbitrary outside observer);

$$p_j(t) = \frac{\hat{\lambda}_0(t)\hat{\lambda}_1(t)\ldots\hat{\lambda}_{j-1}(t)}{\hat{\mu}_1(t)\hat{\mu}_2(t)\ldots\hat{\mu}_j(t)} p_0(t). \tag{8}$$

This means that for each time $t > 0$ the state probabilities can, in principle, be determined subject to specification of the initial conditions $p_0(t)$ (i.e. the so-called absorbing state) and the product of birth–death ratios at all previous states.

To obtain the above transient solution for $p_j(t)$ (i.e. for finite t) in closed form, it is necessary to postulate basic expressions for $\hat{\lambda}_j$ and $\hat{\mu}_j$. Here we assume that—for the case of a finite probability distribution, i.e. $p_j(t) > 0$ (with $j = 1, 2, \ldots, N$)—clusters form via an entropy-like formula

$$\hat{\lambda}_j(t) = -\sum_{k=1}^{M} \lambda_k(t) \ln \lambda_k(t) > 0 \qquad (j = 0, 1, 2, \ldots, N - 1) \tag{9}$$

with $\lambda > 0$ for all k. According to information theory [28], the shape of our birth coefficients may reflect the measure of co-operative information carried by the outcomes $\lambda_1, \ldots, \lambda_M$ (or the amount of disorder in the discrete observable $\hat{\lambda}_j$) in a system of M different species or types (for example, human gender, financial traders).

On the other hand, in analogy to Erlang loss systems [27], we assume the clustering completion rate for the system in state j to be exponentially distributed (with rate $\mu > 0$); hence we deal with simple linear death processes

$$\hat{\mu}_j(t) = j\mu(t) \qquad (j = 1, 2, \ldots, N). \tag{10}$$

Then, the equilibrium state probabilities given by equation (8) become

$$p_j(t) = \frac{(-1)^j}{j!}\left[\sum_{k=1}^{M} a_k \ln \lambda_k(t)\right]^j p_0(t) \qquad (j = 0, 1, 2, \ldots, N) \tag{11}$$

where the *per capita* ratio $a_k \equiv \lambda_k(t)/\mu(t) > 0$ is, for simplicity, assumed to be time independent. This means that λ_k and μ should both scale as power laws of the form $\sim\Delta t^{\pm n}$.

By using the normalization condition of equation (6) plus (11) and the Taylor series expansions for the *exponential* function, we also obtain

$$p_0(t) \equiv \left[\sum_{j=0}^{\infty} \frac{(-1)^j}{j!}\Gamma_M^j(t)\right]^{-1} = \mathrm{e}^{\Gamma_M(t)} \tag{12}$$

where

$$\Gamma_M(t) \equiv \sum_{k=1}^{M} a_k \ln \lambda_k(t) = \ln \prod_{k=1}^{M} \lambda_k^{a_k}(t) = \ln p_0(t). \tag{13}$$

Now that we have a precise formulation for $\tilde{P}_j(t)$, we can set trial functions for the basic interpolation functions g_j to solve for $G_N(s, t)$ given in equation (4).

2.1.2. Trial $g_j(s)$ functions. The efficiency of the Galerkin formulation is very dependent on making the correct choice of the approximating test and trial functions (see the appendix). Of the many nodal unknowns that could be candidates, here we consider the polynomial expansion

$$g_j(s) \equiv \frac{\gamma}{(s-1)^j} = \gamma\left\{(-1)^j + \binom{j}{1}(-1)^{j-1}s + \binom{j}{2}(-1)^{j-2}s^2 + \cdots\right\} \tag{14}$$

with s a dimensionless variable.

Substitution of equation (11) into (5) and using (13) plus these trial functions gives the approximate trial solution

$$G_N(s, t) = \gamma p_0(t) \sum_{j=0}^{N} \frac{1}{(s-1)^j}\left\{\frac{(-1)^{2j}}{(2j)!}\Gamma_M^{2j}(t) + \frac{(-1)^{2j+1}}{(2j+1)!}\Gamma_M^{2j+1}(t)\right\} \tag{15}$$

as is easily verified.

We shall see next that our g_j functions are a judicious choice.

2.2. *Onset of log-periodicity*

Let us consider large systems in statistical equilibrium and adopt the following notation for the required correction term of equation (1) near t_0:

$$G_\infty(t) \equiv \lim_{N\to\infty} G_N(s \approx 0, t). \tag{16}$$

For the sake of simplicity we have set $s \approx 0$ in order to gain insight into the genesis of log-periodicities.

As an example, our approximate trial solution of equation (15) thus becomes

$$G_\infty(t) = \gamma p_0(t)\left\{\sum_{j=0}^{\infty}\frac{(-1)^j}{(2j)!}\Gamma_M^{2j}(t) - \sum_{j=0}^{\infty}\frac{(-1)^j}{(2j+1)!}\Gamma_M^{2j+1}(t)\right\}. \tag{17}$$

Using Taylor series expansions for the *cosine* and *sine* functions plus the trigonometric identity $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$, such that $a = \pi/4$ and $b/2\pi \equiv \Gamma_M(t)$ in radians, the above equation then results in

$$G_\infty(t) = \sqrt{2}\gamma p_0(t)\cos\left(2\pi\ln p_0(t) + \frac{\pi}{4}\right) \tag{18}$$

with p_0 satisfying equation (13).

This equation characterizes the complexity of the underlying dynamics of the scaling systems under consideration. It can be seen that in our stochastic theory the log-periodic modulation is a consequence of the entropy-like assumption used for the transmission of information within the birth–death clustering processes.

Using our expression for the initial boundary condition p_0 leading to equation (18), we analyse next the presence of a singularity at a finite time where the oscillations accumulate.

2.3. Finite-time singularity

As discussed in the derivation of equation (11) μ should scale with t, so we set

$$\mu(t) \sim (t_0 - t)^{\pm n} \tag{19}$$

where $n \neq 0$ is a given exponent and t_0 characterizes the finite-time singularity. Hence, according to our definitions

$$\lambda_k(t) \sim a_k(t_0 - t)^{\pm n} \tag{20}$$

which implies that within our stochastic birth–death theory we can consider finite values of time $t < t_0$ (or n even only if $t > t_0$) since the coefficients λ_k and μ_k (and, therefore, a_k) are all positive.

By substitution of this scaling into equation (13), we finally get

$$p_0(t) = \prod_{k=1}^{M}\lambda_k^{a_k}(t) = \Theta_M\prod_{k=1}^{M}(t_0 - t)^{\pm n a_k} \tag{21}$$

where

$$\Theta_M \equiv \prod_{k=1}^{M} a_k^{a_k}. \tag{22}$$

Thus, from this relation and equation (18), we are able to derive log-periodic corrections in the form of G_∞ for $t < t_0$.

3. Discussion

Having introduced our theory based on stochastic clustering processes to describe log-periodic corrections to scaling and a finite-time singularity at t_0, we use equations (1), (3), (18) and (21) to obtain

$$G(t) = A + B(t_0 - t)^{\alpha} + C(t_0 - t)^{\beta}\cos(2\pi\beta\ln(t_0 - t) + \psi) \tag{23}$$

Table 1.

	A	B	C	α	β	t_0	ψ	rms
S&P500 index	1.51	4.34	−0.01	−0.1	1.42	1988.62	1.85	0.053
World population	0.25	1489.74	−25.24	−1.38	−1.04	2054.61	−6.34	0.047
Seismic activity	−1.38	1.18	−0.04	−0.74	−1.25	1980.29	0.78	0.414

where $A \equiv G_0(t_0)$, $B \equiv -\kappa/\alpha$, and α are parameters relating the pure power law term of the governing equation (1), and

$$C \equiv \sqrt{2}\gamma\Theta_M \qquad \beta \equiv \pm n \sum_{k=1}^{M} a_k \qquad \psi \equiv 2\pi \ln \Theta_M + \frac{\pi}{4} \tag{24}$$

are the parameters of the log-periodic corrections in terms of our stochastic birth–death model parameters. The fit of this equation to historical random data displaying accelerated precursory patterns and a spontaneous singularity, which indicates the sharp transition to a new regime, is presented in figure 1. As illustrative examples, we plot the daily Log(S&P500) stock index closing values during the years 1982–8 [20], the estimated 1000–2000 world population by the UN Population Division [29] and the sum of seismic activities measured near the Virgin Islands between April 1979 and Feb. 1980 [30]. Our best fits with equation (23) to these data sets were performed as given in table 1.

Examination of the plotted curves shows that equation (23) can model log-periodic corrections to the leading scaling behaviour and a singularity at t_0 in different applied domains similarly to the methods inspired by renormalization group theory entailing complex critical exponents. If we set $\alpha = \beta = 1/f$ we can approach the results obtained in [7–9] where market crashes, population explosion and culminating large earthquakes are viewed as critical points in a system with an underlying discrete scale invariance. If we set $\alpha \neq \beta$, our results are then comparable to those obtained using the more general ansatz of two different exponents as in [10]. The main difference of our stochastic theory with respect to the critical exponent approach is the presence of the exponent β also appearing in the argument of the cosine function. Since n and a_k are positive, then from equation (24) we have that $\beta \neq 0$. Furthermore, in our stochastic theory the finite time t_0 is also determined as a function of β, as discussed below.

So far, the fitting in figure 1 allows us to argue that the apparent logarithmic periodicities in scaling systems may also be understood within the context of a stochastic analytical model based on birth–death clustering processes which is the distinctive feature of our stochastic theory. We can interpret the ‘birth’ and the ‘death’ clusterings in different ways: for financial systems the ‘births’ and ‘deaths’ may represent the buyers and the sellers, respectively; for population growth, the newborns and deceases would be in correspondence and absorbed and released energy may relate the ‘births’ and ‘deaths’ in the case of seismic events for finite times $t < t_0$.

The recurrence equations (7) are *conservation-of-flow* relations. That is, the long-run rate at which the system moves up from state j to $j + 1$ equals the rate at which the system moves down from state $j + 1$ to j (i.e. rate up = rate down). Thus, birth–death processes describe the stochastic evolution in time of a random variable whose values varies (i.e. increases or decreases) by one in a single event (or state) starting from the absorbing state p_0.

The spontaneous singularity is here related to the birth–death coefficients which in turn determinate p_0, i.e. the initial boundary condition at the state $j = 0$ via equation (12). It is also important to note that the state distribution coefficient defined by

$$-\left(\frac{p_{j+1} - p_j}{p_j}\right) = 1 + \frac{\Gamma_M}{j+1} \tag{25}$$

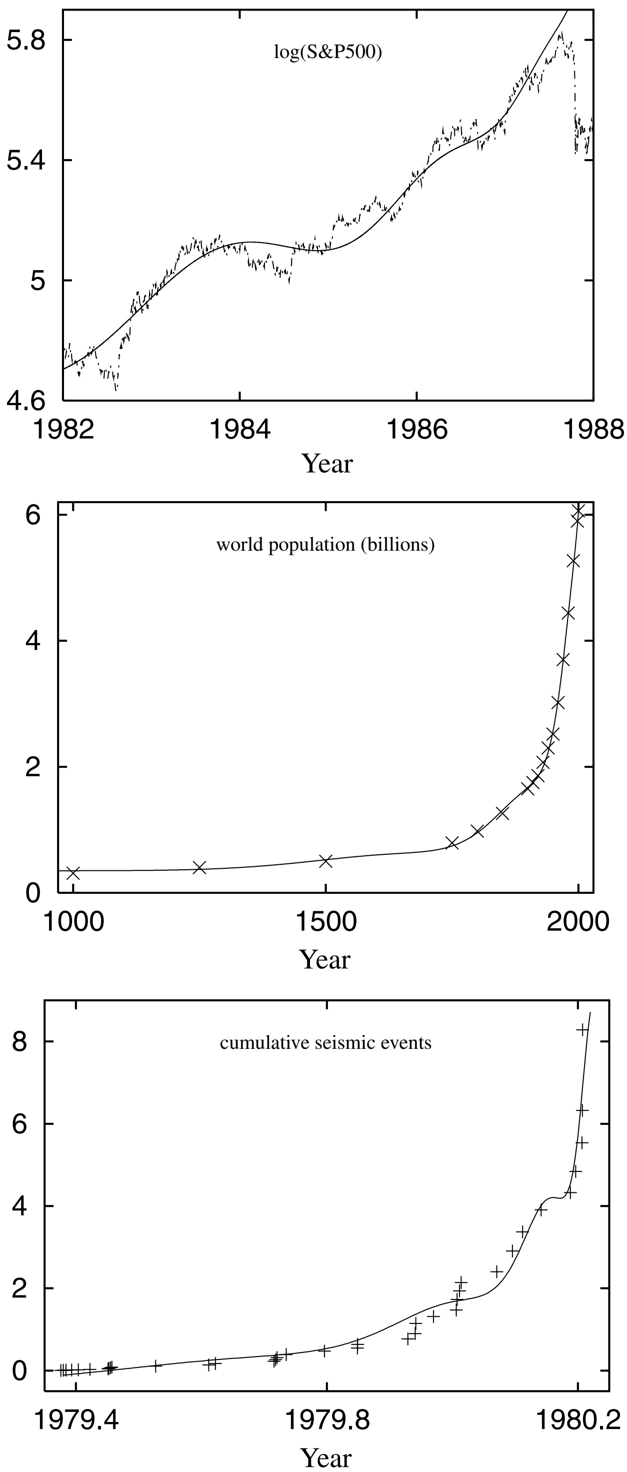

Figure 1. Illustrative examples of log-periodic patterns. Full curves are the fits of our birth–death clustering theory using equation (23). These fits allow us to estimate the total sum of outcomes λ_k.

depends on the logarithm of the absorbing state via equation (13).

In the absence of a well-defined nonlinear dynamical equation governing log-periodic corrections to power law scaling, we have adopted the standard Galerkin finite-element method as the starting point to search for a general trial solution to this 'unknown' differential equation. The motivation for our basic interpolation functions $g_j(s)$ follows computational finite-element methods which are characterized by the use of polynomials for the known test functions (obtained from equations (5) to (13)) as well as for the unknown trial functions of equation (14) in subdomains called finite elements [26].

As discussed in the appendix, our trial $g_j(s)$ allow us to solve the matrix equation for the test functions, which have been related to a large number of (birth–death) processes forming clusters, provided that the governing equation of the problem is known. These forms, using $s \approx 0$, were taken for convenience to gain insight into the onset of log-periodicities. If we consider instead small $s \ll 1$, we would obtain similar conclusions after some algebra.

We thus argue that log-periodicities are a consequence of transient clusters induced by the entropy-like term given in equation (9) which may reflect the amount of co-operative information carried by the state of a large system (i.e. $N \to \infty$) of different species M. Using the definition of the amount of (discrete) finite information or entropy, it can be shown that the information is additive under concatenation of independent probabilities, as is the logarithm function. It has been proved that it is possible to define information without necessarily using the concept of probability (see, for example, [28]). We have adopted the latter definition in this paper via equation (9) for the birth coefficients. The clustering completion rates for the system are given by a simple linear death process.

The state probablities p_j are normalized via equation (6). We may also consider that the total sum of outcomes λ_k is constant for all time as in the Shannon theory of information [28]. Therefore, for the whole range M of different species we set

$$\sum_{k=1}^{M} \lambda_k(t) \equiv \xi_t > 0. \tag{26}$$

We can then estimate the finite time t_0 at which a singularity appears from equations (20) and (24) by considering the initial time $t = 0$ to thus obtain the relation

$$t_0^{\pm n} = \frac{\pm n \xi_0}{\beta}. \tag{27}$$

From the examples in figure 1 we found for the Log(S&P500) stock index $n = 0.105$ and $\xi_0 = 30$, for world population $n = -0.209$ and $\xi_0 = 1$ and for accumulated seismic activity, $n = -0.226$ and $\xi_0 = 1$.

The *per capita* ratios a_k plus the exponent n appearing in the scaling of equation (19) are the minimum ingredients required to derive a complete description of log-periodic corrections to scaling and finite-time singularities within the framework of a stochastic theory based on birth–death clustering processes. The positive state distributions p_j are determined by n and a_k which also relate the exponent β, C and ψ as in equation (23) and t_0 as in (27). This means that such state distributions of the system drive the log-periodic oscillations. We believe that this feature of our stochastic model can help to elaborate a general microscopic theory to understand the underlying mechanisms of log-periodic patterns. In the case of financial systems, such a microscopic theory should also explain the peculiar statistical features in short time scales such as the highly correlated variance or volatility of price fluctuations [31, 32], by exploring the state distribution coefficient given by equation (25).

Appendix. The Galerkin formulation

In this appendix, the key features of the standard Galerkin finite-element method are stated concisely for completeness [26]. If a 2D problem in a domain $D(x, y)$ is governed by a linear differential equation $L(u) = 0$, with boundary conditions $S(u) = 0$ on δD, i.e. the boundary of D, then the Galerkin method assumes that u can be accurately represented by the approximate trial solution

$$u(x, y) = u_0(x, y) + \sum_{j=1}^{N} a_j(y)\phi_j(x) \tag{28}$$

where the ϕ_j are known trial analytical functions, u_0 is chosen to satisfy the boundary conditions and the a_j are test functions to be determined.

To obtain the unknown a_j, the inner product of the weighted residual R is set equal to zero:

$$(R, \phi_k) \equiv \int\int_D R\phi_k \, \mathrm{d}x \, \mathrm{d}y = 0 \qquad k = 1, \ldots, N \tag{29}$$

where

$$R(a_0, a_1 \ldots a_N, x, y) \equiv L(u) = L(u_0) + \sum_{j=1}^{N} a_j(y)L(\phi_j). \tag{30}$$

Since this example is based on a linear $L(u)$, then the above can be rewritten as a matrix equation for the a_j as

$$\sum_{j=1}^{N} a_j(t)L(\phi_j, \phi_k) = -L(u_0, \phi_k). \tag{31}$$

Substitution of the a_j resulting from this equation into (28) gives the required approximate solution $u(x, y)$.

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